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An Aerodynamic Model for A Low-Altitude Rocket Exhaust Plume

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ABSTRACT

An aerodynamic model is proposed for the exhaust plume issuing from a rocket nozzle exit into quiescent ambient air in which chemical reaction or two-phase flow does not predominate. The model is based on the presently available knowledge and established aerodynamic principles for the various flow regimes involved. The major assumptions made are individually substantiated and justified by experimental evidence in existing literature.

A theoretical analysis is carried over the whole flow region based on the Kármán's integral approach in the hope of finding solutions for the gross aerodynamic behavior of the plume, especially for ranges up to the full region of the plume. Solutions are matched at the boundary of the inviscid core and turbulent mixing regions. Theoretical results agree closely with related experimental data.

PROBLEM STATUS

This is an interim report on one phase of the problem; work on this and other phases is continuing.

AUTHORIZATION

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AN AERODYNAMIC MODEL FOR A LOW-ALTITUDE ROCKET EXHAUST PLUME

INTRODUCTION

In recent years, the investigation of the aerodynamic and electrodynamic properties of rocket exhaust plumes has been stimulated by a need for knowledge of their electrical and optical properties. Because the hot gases of the rocket exhaust plume leave the rocket nozzle at supersonic speeds, any disturbance developed in the exhaust plume does not travel upstream to the rocket nozzle to cause aerodynamic effect there. Thus the study of the exhaust plume was not stimulated by rocket thrust considerations. Nevertheless the state of the gas in the plume at the exit plane reflects the energy condition of the plume and thus the efficiency of energy conversion in the rocket nozzle. It also dictates the interactions of the exhaust gas with the ambient air outside the motor, and thus the gas dynamic and plasma properties of the exhaust.

In many rockets, communication links are maintained by transmitting electromagnetic waves which pass through the hot gases in the exhaust plume. Because of the presence of free electrons in the exhaust plume of both liquid and solid propellant rocket motors, the transmitted electromagnetic waves may experience absorption, reflection, refraction and phase shift (1). These processes constitute elements of the undesirable phenomenon of radar attenuation which seriously interferes with tracking and communications operations. Mainly for this reason, the rocket exhaust plume has been of concern in the design of space boosters and missile communication systems. A general survey of this aspect of the problem can be found in such works as those by Smoot, Underwood and Schroeder (2), Rosner (3), Calcote, Kurzius and Silla (4), and Balwanz (5).

Similarly, optical properties are of interest because of tracking and detection capabilities in the visible and the near infrared and ultraviolet regions. Both the electron density distribution and the optical characteristics in a rocket exhaust plume are controlled by a large number of very complex simultaneous and interrelated processes involving many components. These processes in most instances are thermodynamic, chemical kinetic, and electrodynamic as well as aerodynamic in nature. Their gross behavior with respect to electron distribution and radiation in the exhaust plume include the presently identified phenomena of thermal, shock, and afterburning reactions. All of these phenomena are strongly aerodynamically oriented. In particular, the process of afterburning is dominantly controlled by the turbulent mass diffusion of residue fuel molecules in the plume and of oxygen molecules in the fresh ambient air to a common region for combustion. The turbulent mass diffusion process is totally aerodynamic in nature but is complicated both by the two-phase flow prevalent in operational rocket exhaust and, in flight, by turbulences induced in the otherwise quiescent air by the vehicle body. Recent experimental findings on radar attenuation of rocket exhaust plume reported by Balwanz (5) further reveal that in many cases the processes of afterburning and of dilution by entrained air, another totally aerodynamic entry, seem to dominate. Therefore, it can be said that aerodynamics not only provides the basis tools to analyze the dominant processes controlling radar attenuation in a rocket exhaust plume but also serves as a common carriage upon whose structure the other processes can be attached and analyzed.

The difficulties in an aerodynamic analysis of the rocket exhaust plume are usually twofold, one purely aerodynamical and the other belonging to the establishment of a justifiable aerodynamic model of the rocket exhaust plume to its entire extent. The purely aerodynamical difficulties come mostly from such things as the nonlinearity of the governing differential equations and the lack of precise and accurate understanding of the detailed mechanism of turbulent mixing. With the advancement of nonlinear mathematical technique, especially if only gross aerodynamic behavior of the flow field is being investigated, quite a few of the difficulties of this nature have been removed.

In the latest aerodynamic models in the study of rocket exhaust plumes now in the existing literature, the flow field is always first artificially broken down into several pieces which are analyzed independently, and the results from these uncoupled analyses are artificially combined as represented by the analysis by Smoot, Underwood, and Schroeder (2). The validity of these models extends to a region downstream of the rocket nozzle only as far as a distance of the order of the diameter of the nozzle exit at most, which is orders of magnitude smaller than the total range of the rocket exhaust plume. The important processes of afterburning and air dilution usually become severe at axial stations downstream of the rocket exit and its immediate neighboring region. Consequently the establishment of a more realistic and justifiable aerodynamic model of rocket exhaust plume for extended axial ranges, even if necessarily at the sacrifice of detailed accuracy, is in order.

PROPOSED AERODYNAMIC MODEL

General Structure of the Exhaust Plume

Investigations on free subsonic jets (length of core, quantity of entrained air, and decay of velocity) have been made by Kuethel (6), Wuest (7) and Squire (8). Their theoretical results based on the assumption of an incompressible fluid agree closely with experimental findings. Investigations in the literature to date on free supersonic jets issuing into quiescent ambient air have been restricted to a few isolated partial experimental studies. However, these experimental studies reveal a qualitative general picture of the structure of the free supersonic jet.

Frauenberger and Forbister (9) used a pitot tube to measure the total pressures and total temperatures at various positions in the exhaust plume of a nonaluminized solid-propellant rocket motor and computed the velocities of the gas-air mixture from these measurements. Their study was conducted at static thrust and at sea level pressure. The exhaust plume had relatively low afterburning. Their observations are qualitatively summarized in Fig. 1. A jet leaving a nozzle at initially supersonic velocity contains

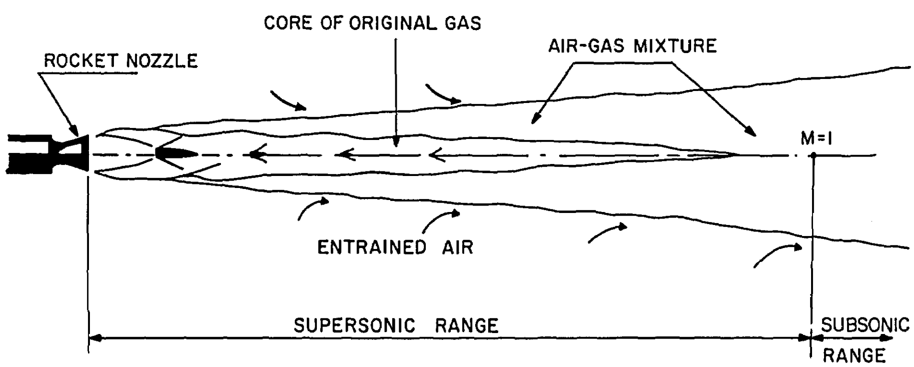

Fig. 1 - Observed general structure of a rocket exhaust plume (after Ref. 9)

along its axis the gradually narrowing core of the original gas stream surrounded by a gradually widening annular region where gas and entrained air have intermixed. As mixing extends across the cross section, temperature and velocity fall, and further mixing of air with gas and already entrained air takes place at decreasing speeds. Their few pressure measurements following paths along and across the axis in the region of the undisturbed supersonic core of original gas also indicate the general cyclic nature of that portion of the flow field.

Anderson and Johns (10) independently measured the spreading and axial decay of free supersonic jets exhausting into quiescent air. Their work involved the measurement of total pressures and temperatures downstream of five supersonic heated-air jets and three types of solid-propellant rockets. The low-combustion-temperature propellants used should have provided exhaust streams with low chemical reactions and be similar to the thermally heated air jets. Their observations in general confirm the sketch of the general structure shown in Fig. 1.

The general structure of the proposed aerodynamic model of a rocket exhaust plume in the light of the existing experimental observations will be assumed to be as shown in Fig. 2. The supersonic gas stream leaving a rocket nozzle remains undisturbed within a linearly decreasing straight cylindrical cone of base diameter $2R$ and length L. The effective plume radius R near the nozzle exit is usually the radius of the nozzle exit but may deviate appreciably in many cases. Both R and the length of the undisturbed supersonic core L would have to be determined either from a theoretical calculation or experimentally. Surrounding the undisturbed supersonic core is an annular region of turbulent mixing between the rocket exhaust gas and ambient fresh air and of the resulting afterburning. The r-coordinate or radial thickness of the turbulent mixing region increases with the x-coordinate or axial distance from the nozzle exit plane. The original undisturbed supersonic gas disappears completely at $x = L$. The axial velocity at the center of the plume remains supersonic beyond $x = L$ but diminishes, in value, reaching sonic velocity at $x = x_s$. Thus the flow field beyond $x = x_s$ is fully subsonic.

Fully Subsonic Turbulent Flow Region $(x \geq x_s)$

For the fully subsonic turbulent flow region, experimental results on the lateral spreading of the plume and the axial decays of temperature and velocity by Frauenberger

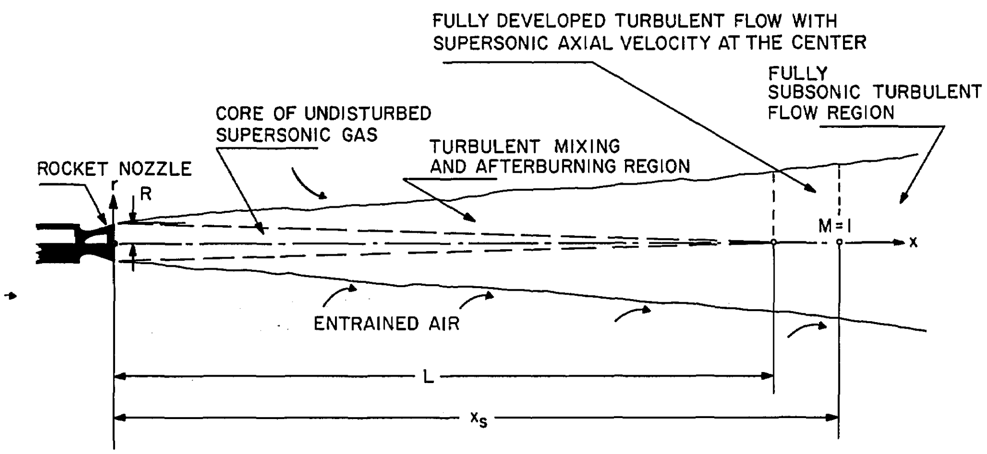

Fig. 2 - General structure of the proposed aerodynamic model of a rocket exhaust plume

and Forbister (9) and Anderson and Johns (10) check closely with the existing classical theories for a fully developed turbulent incompressible jet issuing into a still ambient fluid of approximately the same density. The axial velocity profiles at various axial distances have been repeatedly found to be very nearly Gaussian and reduce to similar shape when properly normalized. The static pressure field has been repeatedly found to be practically the same as the constant static pressure of the ambient air.

Details of the subsonic turbulent flow region of the proposed aerodynamic model of a rocket exhaust plume are shown in Fig. 3. The pressure field P and the density field ρ are assumed to be constant throughout this region and equal to the pressure P_0 and the density ρ_0 of the ambient still air. The time-mean axial velocity field $u(x, r)$ is assumed to have the Gaussian distribution

$$u(x,r) = u(x) \exp(-r^2/b^2), \tag{1}$$

where $u(x)$ is the time-mean axial velocity at a point x along the x axis and at the same point along the x axis $b = b(x)$ is a characteristic radius at which the local time-mean axial velocity assumes a magnitude $1/e$ (1/2.718...) times the time-mean axial velocity. At $x = x_s$ the assumed axial velocity distribution becomes

$$u(x_s, r) = u_s \exp(-r^2/b_s^2), \tag{2}$$

where u_s is the local sonic velocity along the axis and the characteristic radius $b_s = b(x_s)$ would have to be computed from flow properties in earlier regions ($x \leq x_s$).

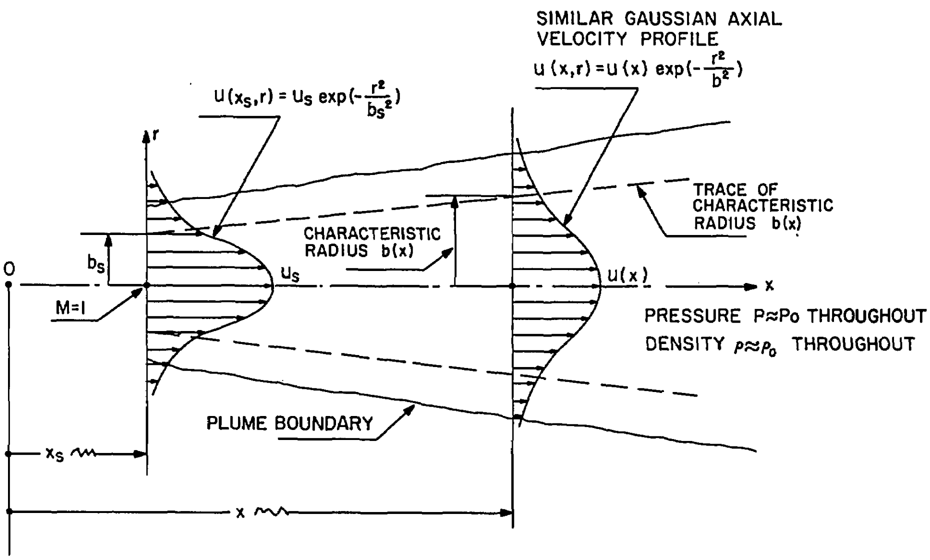

Fig. 3 - Details of the subsonic turbulent flow region of the proposed aerodynamic model of a rocket exhaust plume

Region of the Undisturbed Supersonic Core of Original Gas $[0 \leq x \leq L,\ r \leq r_1(x)]$

For a parallel supersonic jet leaving a slightly underexpanded two-dimensional nozzle and expanding into still air without any intermixing between the two fluids at their common boundary, Prandtl (11) gives a wave diagram as shown in Fig. 4. In this idealized situation the jet, as it flows from the nozzle, expands and contracts between the pressures P_e and P_0' in such a way that

$$P_e/P_0 = P_0/P_0' , \tag{3}$$

where P_e and P_0 are the pressures of jet at nozzle exit and of ambient still air respectively and P_0' is the pressure of the diamond-shaped expansion regions of the jet. The periodical expansion-contraction pattern of the supersonic jet is characterized by wavelengths

$$\lambda_i = \frac{\pi}{c_i} \bar{d} \left(\frac{u^2}{u_s^2} - 1 \right)^{1/2} , \tag{4}$$

where c_i is the ith root of the Bessel function of the zeroth order, $J_0(c)$, $\bar{d}$ is the characteristic width of the supersonic jet, u is the jet velocity, and u_s is the local velocity of sound in the jet. Assuming that the major contribution to the wave pattern is made by the first root of $J_0(c)$, where $c_1 = 2.405$, one obtains

$$\lambda = 1.307\, \bar{d} \left(\frac{u^2}{u_s^2} - 1 \right)^{1/2} = 2.614\, \bar{r}\, (M^2 - 1)^{1/2} , \tag{5}$$

where λ is the dominant wavelength, $\bar{r}$ is the characteristic half-width of the supersonic jet, and $M = u/u_s$ is the local Mach number of the jet.

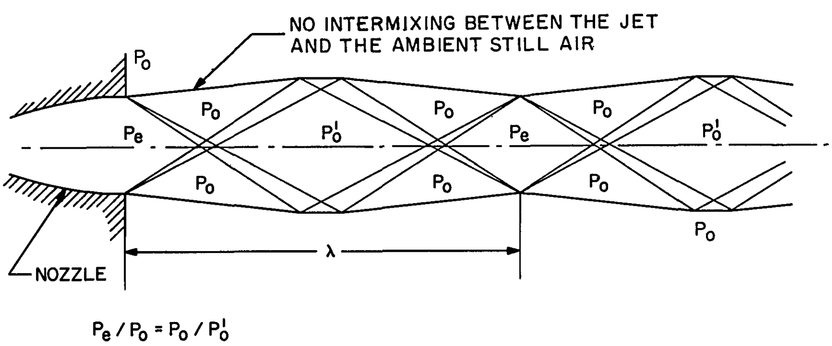

Fig. 4 - Wave diagram of a supersonic jet from an underexpanded parallel nozzle without mixing with the ambient fluid (after Ref. 11)

The flow from a slightly underexpanded axisymmetrical nozzle will also have alternate regions of high and low pressure with a more complex but similar pattern of alternate expansion and compression waves. Therefore, the expression for the dominant wavelength for the axisymmetrical case should be expected to be essentially the same as that for the two-dimensional case given above, perhaps with a slightly different multiplying constant. Altman (12) investigated photographically the wave pattern of the exhaust plume of an axisymmetrical 50-pound-thrust liquid-propellant rocket motor. His measured initial wavelengths in most cases of various experiments, surprisingly enough, agree closely with values computed from Prandtl's expression for the dominant wavelength for the two-dimensional case, the half-width in Eq. (5) being replaced by the radius of the rocket nozzle at the exit.

Caution must be used in interpreting the results obtained with such photography of visible wave patterns. The structure predicted by the Prandtl development is visible in schlieren photography but does not always correspond to the structure visible in normal photography. Chemical reactivity induced by the changing pressure and temperature in the supersonic flow regions frequently causes radiation, resulting in the visible structure in the plume. But reaction delays may cause a shift downstream of the radiating boundaries and the difference between the shock boundaries and the radiating pattern may be significantly large in many cases. Thus the fact that no structure is visible does not mean that the Prandtl structure is not present, nor does a visible structure necessarily follow the Prandtl pressure boundaries.

Figure 5 shows details of the undisturbed supersonic core of original gas of a rocket exhaust issuing into still ambient air and producing a turbulent mixing region enveloping the undisturbed core. The radius of the undisturbed supersonic core $r_1(x)$ is assumed to decrease linearly with the axial distance x in the fashion,

$$r_1(x) = R(1 - x/L), \tag{6}$$

where R is the effective plume radius near the rocket nozzle exit and L is the length of undisturbed supersonic core. For cases where the pressure of jet at the nozzle exit P_e differs not substantially from the ambient pressure P_0, the axial velocity of the jet in the undisturbed core can be assumed to be essentially constant ($u \approx u_1$) and the density can also be assumed to be essentially constant ($\rho \approx \rho_1$) throughout the whole region of the undisturbed core. The wavelengths are assumed to be computable from

$$\lambda_n = 2.614\ [r_1(x_n)]\ (M_e^2 - 1)^{1/2} \qquad (n = 1, 2, 3, \ldots), \tag{7}$$

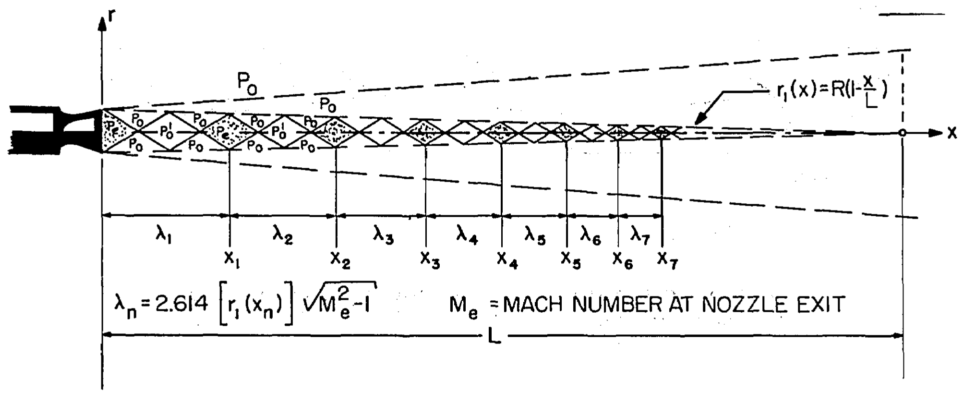

Fig. 5 - Details of the undisturbed supersonic core region of the proposed aerodynamic model of a rocket exhaust plume. The velocity and density are considered to be constant throughout the core region ($u = u_1$ and $\rho = \rho_1$).

where λ_n is the distance between centers of adjacent diamond-shaped compression regions at axial distances $x = x_{n-1}$ and $x = x_n$, $M_e = u_1/(u_s)_e$ is the Mach number of the jet at the nozzle exit, and $(u_s)_e$ is the local velocity of sound in the jet at the nozzle exit. With the assumed form of $r_1(x_n)$ introduced, the above expression can be rearranged to give

$$\frac{\lambda_n}{R} = \left(1 - h\,\frac{R}{L}\right)^{n-1} h \qquad (n = 1, 2, 3, \ldots), \tag{8}$$

where $h = 2.614\ (M_e^2 - 1)^{1/2}$. It can be noted that for a representative plume with $M_e = 3$, the location of the seventh node is computed to be $x = 3L/4$ which generally agrees with many experimental observations made with rocket exhaust plumes. In this portion of the exhaust plume it has been assumed that the pressure of the plume at the nozzle exit P_e is not very different from the pressure of the ambient air P_0. The boundary of the undisturbed supersonic core would then be fairly smooth, and the velocity would be nearly axial and nearly equal to the velocity at the nozzle exit. The assumptions of a straight boundary of undisturbed core and uniform supersonic velocity would under these circumstances seem justified. If, however, P_e is very different from P_0, the boundary of the undisturbed core would then become wavy, with the expansion and contraction regions showing up. Even for that case a smooth average boundary surface can still be assumed. Then all the wavelengths can probably be computed by the proposed scheme except, perhaps, the first one which, however, can readily be computed from the nozzle angle, exit pressure ratio, etc., as perfected in existing literature such as the work by Smoot, Underwood, and Schroeder (2).

Annular Turbulent Mixing and Afterburning Region $[0 \leq x \leq L,\ r \geq r_1(x)]$

A free-jet boundary originates at the point where two parallel streams of different velocity meet. The velocity discontinuity results in the formation of a mixing zone which grows in width in the downstream direction. At sufficiently high Reynolds number, the mixing becomes turbulent. Maydew and Reed (13) experimentally investigated the turbulent mixing region of axisymmetrical supersonic room-temperature air jets (M_e = 1.49 and 1.96) issuing into quiescent ambient air. They found a radially annular turbulent mixing region formed around the undisturbed core of original supersonic air with very nearly linearly decreasing radius with axial distance as shown in Fig. 6. The normalized nondimensional axial velocity distributions were found to be similar and very nearly Gaussian in shape, and correlate very well with turbulent mixing theory for incompressible boundary layer flow. The thickness of the annular mixing region was found to spread

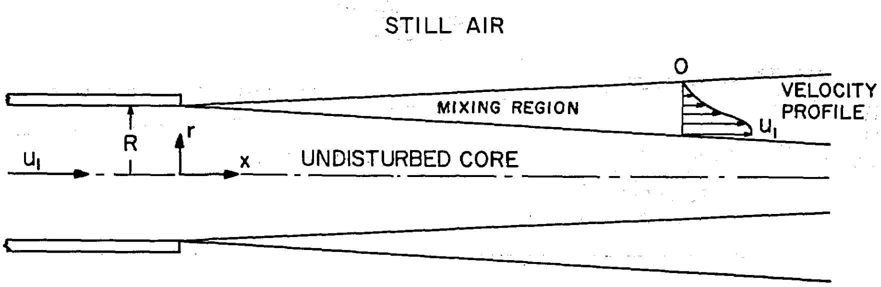

Fig. 6 - Observed turbulent mixing region around the undisturbed core of an axisymmetric supersonic air jet (after Ref. 13)

very nearly linearly with the axial distance from the nozzle exit. The slightly varying pressure field was found to have negligible effect on the velocity field. Smoot, Underwood, and Schroeder (2) studied theoretically the flow field in the annular mixing region with afterburning. Their approach belongs to the type which considers the inviscid core and turbulent mixing regions separately without relationship to each other. Consequently their result, as far as the overall aerodynamic structure is concerned, cannot be taken very seriously in regions of the plume downstream from the immediate neighborhood of the exit plane. However, in the immediate neighborhood of the exit plane in which their model can be considered approximately valid, their results bring up a somewhat interesting point. Although the velocity and the temperature are found to vary enormously across the mixing layer, the mass density is found to remain essentially constant throughout the whole mixing layer except a negligibly thin sublayer within the mixing layer next to the ambient air in which the mass density adjusts itself from a value close to the density of undisturbed supersonic core gas to that of the ambient air. The probable explanation for this peculiar situation is that the flame zone near the outside edge of the mixing layer is a constant heat source that contributes to the expansion of the fluid in that part of the mixing layer in such manner as to make the mass density practically constant throughout most of the mixing layer.

The details of the turbulent mixing region of the proposed aerodynamic model of a rocket exhaust plume are shown in Fig. 7. The mass density ρ is assumed to be constant throughout the mixing layer and equal to the mean mass density of the gas in the undisturbed supersonic core ρ_1. The pressure P is assumed to be constant throughout the mixing layer and equal to the pressure of the ambient air P_0. The axial velocity distributions $u(x, r)$ across the mixing layer at different axial stations are assumed to be of the similar Gaussian profile

$$u(x, r) = u_1 \exp\left[-(r-r_1)^2/b^2\right], \tag{9}$$

where u_1 is the velocity of gas in the undisturbed supersonic core, $r_1 \equiv r_1(x)$ is the assumed inside edge of mixing layer, and $b \equiv b(x)$ is the characteristic mixing layer thickness. At the end of the mixing layer region ($x = L$), the axial velocity distribution becomes

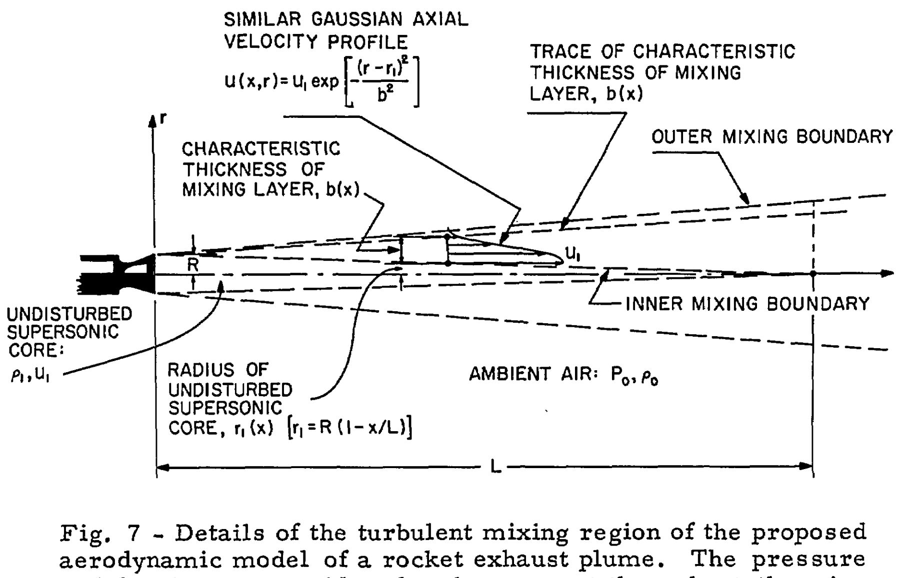

Fig. 7 - Details of the turbulent mixing region of the proposed aerodynamic model of a rocket exhaust plume. The pressure and density are considered to be constant throughout the mixing region ($P = P_0$ and $\rho = \rho_1$).

$$u(L,r) = u_1 \exp(-r^2/b_1^2), \tag{10}$$

where $b_1 = b(L)$, which itself has the form of a similar Gaussian axial velocity profile for a fully developed turbulent flow.

Region of Fully Developed Turbulent Flow With Supersonic Axial Velocity Along the Axis $(L \le x \le x_s)$

No attempt has ever been recorded in the literature to investigate the aerodynamic properties of the gas in the transition region of the rocket exhaust plume between $x = L$ and $x = x_s$ (Fig. 2). However, several generalizations can be made of the aerodynamic properties of the two well-understood neighboring regions, namely, the fully developed turbulent subsonic region $x \ge x_s$ and the turbulent mixing region before the total disappearance of the undisturbed supersonic core. From our established model for the fully developed turbulent subsonic region we have at $x = x_s$ the Gaussian axial velocity profile

$$u(x_s,r) = u_s \exp(-r^2/b_s^2), \tag{11}$$

and the pressure and the density across the plume at $x = x_s$ are both constant and equal to the pressure and the density of the ambient air P_0 and ρ_0 respectively. On the other hand, from our established model for the turbulent mixing region before the total disappearance of the undisturbed supersonic core, we have at $x = L$ the Gaussian axial velocity profile

$$u(L,r) = u_1 \exp(-r^2/b_1^2), \tag{12}$$

and the pressure and the density across the plume at $x = L$ are both constant, with the pressure equal to the pressure P_0 of the ambient air and the density equal to the density ρ_1 of the gas in the undisturbed supersonic core. It then seems reasonable to assume a constant pressure field $P = P_0$, the pressure of the ambient air, for this whole region, to assume a uniform density profile $\rho = \rho(x)$ across the plume at any axial position, where $\rho_1 \le \rho(x) \le \rho_0$, and to assume a similar Gaussian axial velocity profile

$$u(x,r) = u(x) \exp(-r^2/b^2), \tag{13}$$

where $u(x)$ is the axial velocity along the axis, $u_1 \ge u(x) \ge u_s$, and $b \equiv b(x)$ is the characteristic radius at which $u(x,b) = u(x)/e$.

The physical interpretations of this situation may be that the afterburning from the turbulent mixing region is carried into this region in gradually diminishing strength due to a cutoff of fresh fuel supply at the termination of the undisturbed supersonic core of original gas and that in the meantime the aerodynamic properties of the plume have themselves adjusted to values corresponding to the axial station at which the velocity along the axis becomes sonic. For simplicity, the density $\rho(x)$ will be assumed to vary linearly with axial distance as follows:

$$\rho(x) = (\rho_0 - \rho_1)(x - L)/(x_s - L) + \rho_1, \tag{14}$$

which reduces to ρ_1 and ρ_0 at axial distances $x = L$ and $x = x_s$ respectively. A plot of the model is illustrated in Fig. 8.

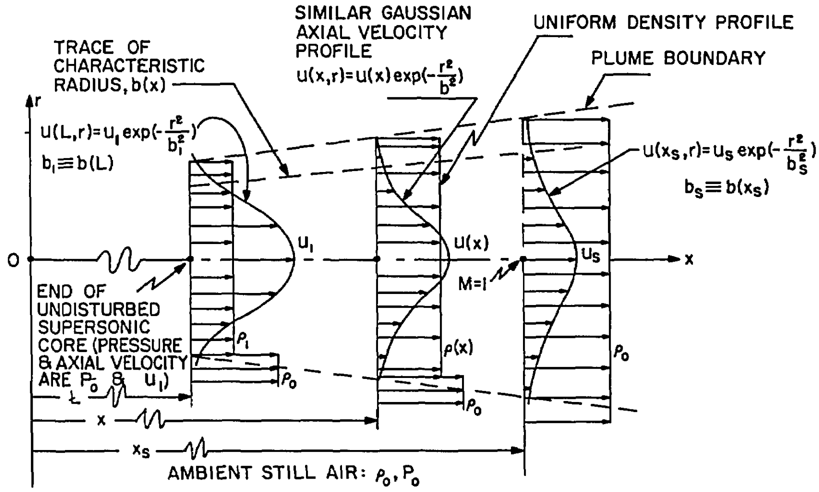

Fig. 8 - Details of the region between the end of undisturbed supersonic core and the beginning of axial subsonic flow ($L \leq x \leq x_s$) of the proposed aerodynamic model of a rocket exhaust plume. The pressure in the region is assumed constant at $P = P_0$, and the density is assumed to be $\rho(x) = (\rho_0 - \rho_1)(x - L)/(x_s - L) + \rho_1$.

THEORETICAL ANALYSIS BASED ON THE PROPOSED AERODYNAMIC MODEL

General Formulation

In both the region of turbulent mixing layer, $0 \leq x \leq L$ and $r \geq r_1(x)$, and the region of totally turbulent flow, $x \geq L$, it will be assumed the flow is fully developed and the usual boundary layer assumptions can be made. Then the molecular transfer mechanism of linear momentum can be negligible as compared to the turbulent transfer mechanism of linear momentum, and the variation of flow properties along the axial direction can be negligible as compared to that along the radial direction. The governing differential equations reduce to the continuity equation and the axial momentum equation for turbulent boundary layer flow. Kármán's integral method can be introduced to these equations. In fully turbulent regions integration of the momentum equation is carried out throughout the total cross section at any axial station, with the continuity equation employed. In the region before the total disappearance of the undisturbed supersonic core of original gas, integration of the turbulent boundary layer momentum equation is carried out throughout the annular turbulent mixing region, and integration of the inviscid momentum equation is carried out throughout the circular cross section of the undisturbed supersonic core of original gas, with the continuity equation employed for both integrations. The resulting equation for either situation takes the simple form

$$d\overline{M}/dx = 0, \tag{15}$$

where $\overline{M}$ is the total axial momentum flux across a cross section at any axial station. This is a statement of the conservation of total axial momentum flux along the axial direction.

Integration of the continuity equation throughout a cross section of the plume at any axial station gives the simple result

$$d\overline{Q}/dx = -2\pi \rho_{edge}\, r_{edge}\, v_{edge}, \tag{16}$$

where $\overline{Q}$ is the total mass flux across a cross section at any axial station, ρ_{edge} is the density of entrained fluid at edge of the plume, r_{edge} is the radius of the edge of the plume, and v_{edge} is the radial entrainment velocity of ambient air at the edge of the plume. This equation essentially states that the increase or mass flux in the axial direction is caused by the radial entrainment of the mass of the ambient air.

Region Preceding the Total Disappearance of the Undisturbed Supersonic Core of Original Gas $(0 \le x \le L)$

For the undisturbed supersonic core, $r \le r_1(x)$, where (Fig. 5) $r_1(x) = R(1 - x/L)$, the axial velocity is $u(x, r) \approx u_1$, the density is $\rho(x, r) \approx \rho_1$, and the pressure adjacent to the mixing region is P_0.

For the annular turbulent mixing layer region, $r \ge r_1(x)$, as given in Fig. 8 the axial velocity is $u(x, r) = u_1 \exp[-(r - r_1)^2/b^2]$, the density is $\rho(x, r) = \rho_1$, and the pressure is P_0. The total momentum flux is then

$$\overline{M} = \int_0^{r_1} \rho_1 u_1^2 \, 2\pi r \, dr + \int_{r_1}^{\infty} \rho_1 u_1^2 \exp[-2(r - r_1)^2/b^2] \, 2\pi r \, dr$$

$$= \pi\rho_1 r_1^2 u_1^2 + \pi\rho_1 u_1^2 \left(\frac{b^2}{2} + \frac{\sqrt{\pi}}{\sqrt{2}} r_1 b\right)$$

$$= \pi\rho_1 u_1^2 \left[R^2 \left(1 - \frac{x}{L}\right)^2 + \frac{b^2}{2} + \frac{\sqrt{\pi}}{\sqrt{2}} - R\left(1 - \frac{x}{L}\right) b\right], \tag{17}$$

since $r_1(x) = R(1 - x/L)$. The momentum flux equation $dM/dx = 0$ requires that

$$-\frac{2R^2}{L}\left(1 - \frac{x}{L}\right) + b\frac{db}{dx} + \frac{\sqrt{\pi}}{\sqrt{2}} R\left[-\frac{b}{L} + \left(1 - \frac{x}{L}\right)\frac{db}{dx}\right] = 0. \tag{18}$$

Let $b(x) = k_1 x + k_2 x^2 + \ldots$. Therefore,

$$-\frac{2R^2}{L}\left(1 - \frac{x}{L}\right) + (k_1^2 x + 3k_1 k_2 x^2 + \cdots)$$

$$+ \frac{\sqrt{\pi}}{\sqrt{2}} R\left[\left(-\frac{1}{L}\right)(k_1 x + k_2 x^2 + \cdots) + \left(1 - \frac{x}{L}\right)(k_1 + 2k_2 x + \cdots)\right] = 0. \tag{19}$$

By putting the sum of coefficients of terms of equal powers of x to zero, we have

$$k_1 = \frac{2\sqrt{2}}{\sqrt{\pi}} \frac{R}{L}$$

$$k_2 = \frac{\sqrt{2}}{\sqrt{\pi}} \frac{R}{L^2}\left(1 - \frac{4}{\pi}\right)$$

$$\cdots$$

Therefore,

$$b(x) = \frac{2\sqrt{2}}{\sqrt{\pi}}\frac{R}{L}x + \frac{\sqrt{2}}{\sqrt{\pi}}\frac{R}{L^2}\left(1 - \frac{4}{\pi}\right)x^2 + \cdots \tag{20}$$

or, in nondimensional form,

$$\left[\frac{b(x)}{R}\right] = \frac{2\sqrt{2}}{\sqrt{\pi}}\left(\frac{x}{L}\right) + \frac{\sqrt{2}}{\sqrt{\pi}}\left(1 - \frac{4}{\pi}\right)\left(\frac{x}{L}\right)^2 + \cdots, \tag{21}$$

a plot of which is shown in Fig. 9. It should be noted that this series solution for $b(x)$ converges very rapidly. The maximum magnitude of the ratio of the second term to the first term is only

$$\left|\frac{k_2 L}{k_1}\right| = 0.137 \;;$$

therefore the first term in the series dominates.

The total mass flux is

$$\begin{aligned}\overline{Q} &= \int_0^{r_1} \rho_1 u_1 2\pi r \, dr + \int_{r_1}^{\infty} \rho_1 u_1 \exp\left[-(r - r_1)^2/b^2\right] 2\pi r \, dr \\ &= \pi \rho_1 u_1 \left(r_1^2 + b^2 + \sqrt{\pi}\, r_1 b\right) \\ &= \pi \rho_1 u_1 \left[R^2\left(1 - \frac{x}{L}\right)^2 + b^2 + \sqrt{\pi}\, R\left(1 - \frac{x}{L}\right) b\right],\end{aligned} \tag{22}$$

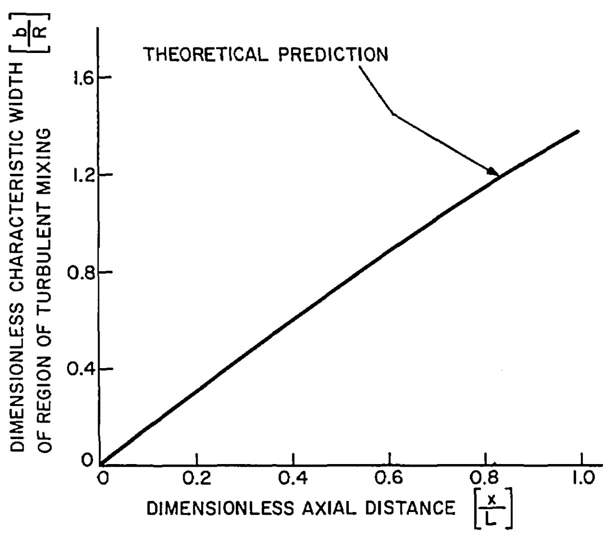

Fig. 9 - Theoretical prediction of the growth of the region of turbulent mixing around a supersonic plume based on the proposed aerodynamic model

since $r_1(x) = R(1 - x/L)$. The mass flux equation can be approximated by

$$\frac{d\bar{Q}}{dx} = -2\pi\rho_0(r_1 + b)\, v_{(r_1+b)} \, . \tag{23}$$

Therefore, the entrainment velocity of ambient air at the characteristic radius $r = r_1 + b$ is then

$$v_{(r_1+b)} = -\frac{1}{2\pi\rho_0}\frac{dQ/dx}{(r_1 + b)}$$

$$= (1 - \sqrt{2})\,\frac{\rho_1 u_1}{\rho_0}\,\frac{R}{L} + \left[\frac{-2\sqrt{2}}{\sqrt{\pi}} + \frac{4}{\sqrt{\pi}} - \frac{8}{\pi} + \frac{4\sqrt{2}}{\pi}\right]\frac{\rho_1 u_1}{\rho_0}\,\frac{R}{L^2}\,x + \cdots \tag{24}$$

or, in nondimensional form,

$$\left[-\frac{\rho_0}{\rho_1}\,\frac{v_{(r_1+b)}}{u_1}\right] = \left[(1 - \sqrt{2})\left(\frac{R}{L}\right)\right] + \left[\left(-\frac{2\sqrt{2}}{\sqrt{\pi}} + \frac{4}{\sqrt{\pi}} - \frac{8}{\pi} + \frac{4\sqrt{2}}{\pi}\right)\left(\frac{R}{L}\right)\right]\left(\frac{x}{L}\right) + \cdots \, . \tag{25}$$

Again, it can be seen that this series also converges very rapidly and therefore that the first term dominates.

The summary of the results for $0 \le x \le L$ are as follows:

$$u(x, r) = u_1 \exp\left[-(r - r_1)^2/b^2\right] \qquad \text{for } r \ge r_1(x),$$

$$u(x, r) = u_1 \qquad \text{for } r \le r_1(x),$$

$$r_1(x) = R(1 - x/L),$$

$$b(x) = \frac{2\sqrt{2}}{\sqrt{\pi}}\,\frac{R}{L}\,x + \frac{\sqrt{2}}{\sqrt{\pi}}\,\frac{R}{L^2}\left(1 - \frac{4}{\pi}\right)x^2 + \cdots ,$$

and

$$v_{(r_1+b)} = (1 - \sqrt{2})\,\frac{\rho_1 u_1}{\rho_0}\,\frac{R}{L} + \left(\frac{-2\sqrt{2}}{\sqrt{\pi}} + \frac{4}{\sqrt{\pi}} - \frac{8}{\pi} + \frac{4\sqrt{2}}{\pi}\right)\frac{\rho_1 u_1}{\rho_0}\,\frac{R}{L^2}\,x + \cdots .$$

Maydew and Reed (13) measured the width of the annular turbulent mixing region of axisymmetrical supersonic room-temperature air jets (M_e = 1.49 and 1.96, where M_e is the Mach number of the jet at the nozzle exit) issuing into quiescent ambient air. The results of Maydew and Reed ($M.\ \&\ R.$) are $(b_{0.1})_{M.\&R.} = 0.087\,x$ for the $M_e = 1.49$ jet and $(b_{0.1})_{M.\&R.} = 0.068\,x$ for the $M_e = 1.96$ jet, where $(b_{0.1})_{M.\&R.}$ stands for a characteristic width of mixing region defined in their notation as the distance between stream lines $u/u_1 = \sqrt{0.10} = 0.3165$ and $u/u_1 = \sqrt{0.90} = 0.95$. In the present notation, it can be easily established that

$$(b_{0.1})_{M.\&R.} = r_{0.3165} - r_{0.95} = 1.072\,b(x) - 0.225\,b(x) = 0.874\,b(x) ,$$

where b is the characteristic width defined in this treatment. Therefore, their experimental results expressed in terms of the present notation become $b(x) = 0.0997\,x$ for the $M_e = 1.49$ jet and $b(x) = 0.0778\,x$ for the $M_e = 1.96$ jet. Comparing these results with our

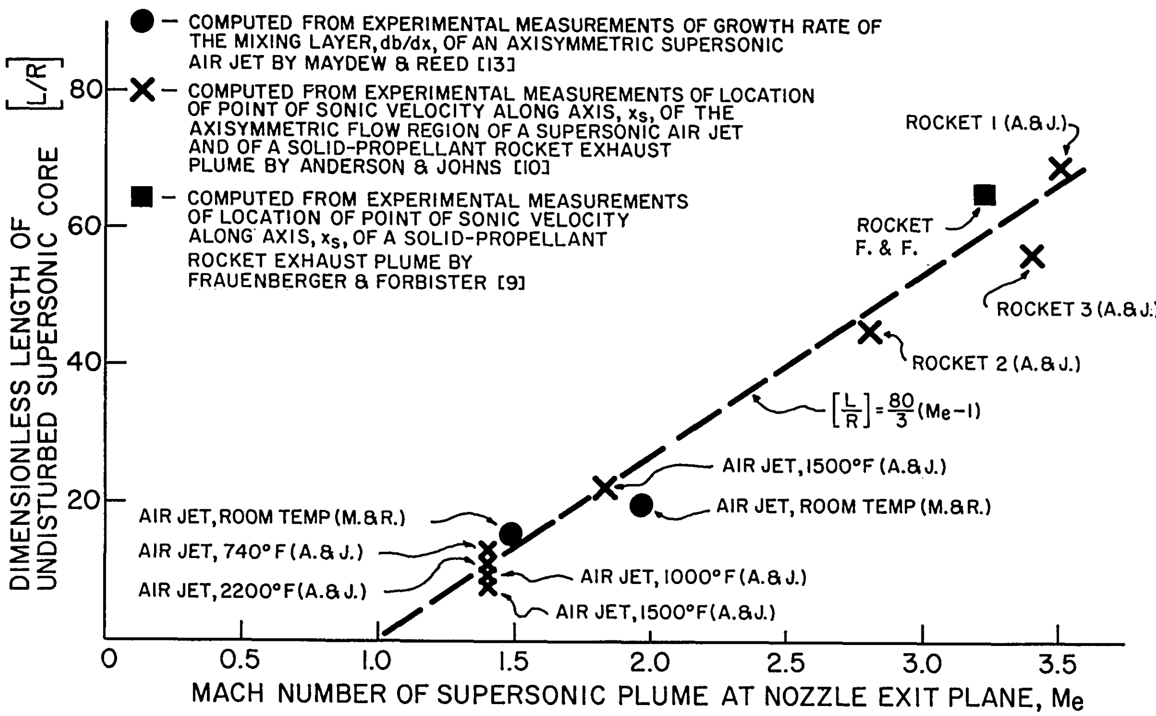

Fig. 10 - Length of the undisturbed supersonic core of an exhaust plume computed by comparing existing experimental measurement on various aerodynamic properties with corresponding theoretical results based on the proposed aerodynamic model

theoretical result

$$b(x) \approx \left(\frac{2\sqrt{2}}{\sqrt{\pi}} \frac{R}{L}\right) x$$

with terms of orders higher than one in variable x dropped, we obtain $L/R = 15.5$ and 19.9 for $M_e = 1.49$ and 1.96 jets respectively, which are plotted accordingly in Fig. 10.

Region of Fully Developed Turbulent Flow With Supersonic Axial Velocity Along the Axis $(L \leq x \leq x_s)$

As given in Fig. 8, the aerodynamic properties for flow in the region $L \leq x \leq x_s$ are as follows: the axial velocity is

$$u(x, r) = u(x) \exp(-r^2/b^2)$$

where $u(L) = u_1$, $b(L) = b_1$, $u(x_s) = u_s$, and $b(x_s) = b_s$; the density is

$$\rho(x, r) = \rho(x) = (\rho_0 - \rho_1)(x - L)/(x_s - L) + \rho_1;$$

and the pressure is P_0.

The total momentum flux is then

$$\overline{M} = \int_0^\infty \rho(x)\, u^2(x) \exp(-2r^2/b^2)\; 2\pi r\, dr$$

$$= \frac{\pi}{2}\, \rho(x)\, u^2(x)\, b^2(x) = \frac{\pi}{2}\, \rho u^2/b^2 \,.$$

The momentum flux equation $d\overline{M}/dx = 0$ requires that

$$\rho u^2 b^2 = \rho_1 u_1^2 b_1^2 = \rho_0\, u_s^2 b_s^2 \,. \tag{26}$$

The total mass flux is

$$\overline{Q} = \int_0^\infty \rho(x)\, u(x) \exp(-r^2/b^2)\; 2\pi r\, dr$$

$$= \pi\rho(x)\, u(x)\, b^2(x) = \pi \rho u b^2 \,. \tag{27}$$

The mass flux equation can be approximated by

$$\frac{d\overline{Q}}{dx} = -2\pi\rho_0\, b v_b \,, \tag{28}$$

where v_b is entrainment velocity of ambient air at the characteristic radius $b(x)$. At this stage, further assumption would have to be made on the nature of the entrainment velocity v_b. Use will be made of an assumption first introduced by Taylor (14) on the lateral entrainment of ambient fluid to an axisymmetrical turbulent free-convection plume caused by the buoyancy force due to temperature difference. His entrainment assumption deals strictly with the velocity field and, aside from minor difference due to a different axial velocity profile he assumed, it essentially takes the form

$$v_b \equiv v(x, b) = -\alpha u(x), \tag{29}$$

where α is the entrainment coefficient, whose value depends solely on the geometry of the problem and the shape of the axial velocity profile assumed. With a Gaussian velocity profile assumed, a value $\alpha = 0.08$ for an axisymmetrical geometry and a value $\alpha = 0.16$ for a two-dimensional geometry have been firmly established by theoretical and experimental works by Morton, Taylor, and Turner (15), Morton (16,17), Rouse, Yih, and Humphreys (18), Lee and Emmons (19), and Lee (20-26). The value $\alpha = 0.08$ for an axisymmetrical geometry has been confirmed by a comparison of solutions based on the entrainment assumption and experimental results on axisymmetrical turbulent incompressible jets by Zimm (27), Ruden (28), Reichardt (29) and Wuest (7).

Introducing Eqs. (27) and (29) in Eq. (28), we have

$$\frac{d}{dx}(\rho u b^2) = 2\alpha\rho_0 u b \,. \tag{30}$$

Introducing Eq. (26) in Eq. (30) with the assumed density $\rho(x)$ used, we have

$$\frac{d}{dx}\left(\frac{1}{u}\right) = \frac{c_2}{[c_3(x - L) + \rho_1]^{1/2}} \,, \tag{31}$$

where $c_2 = 2\alpha\rho_0/\rho_1^{1/2}u_1 b_1$ and $c_3 = (\rho_0 - \rho_1)/(x_s - L)$. Therefore,

$$\frac{1}{u} = \frac{2c_2}{c_3}\,[c_3(x - L) + \rho_1]^{1/2} + c_4\,.$$

Applying the condition $u(L) = u_1$, we have

$$c_4 = \frac{1}{u_1} - \frac{2c_2}{c_3}\,\rho_1^{1/2}\,;$$

therefore,

$$\frac{1}{u} - \frac{1}{u_1} = \frac{2c_2}{c_3}\,\left\{[c_3(x - L) + \rho_1]^{1/2} - \rho_1^{1/2}\right\}$$

$$= \frac{4\alpha}{u_1 b_1}\left(\frac{x_s - L}{1 - \rho_1/\rho_0}\right)\left\{\left[\left(\frac{\rho_0/\rho_1 - 1}{x_s - L}\right)(x - L) + 1\right]^{1/2} - 1\right\}. \tag{32}$$

Substituting Eq. (32) in Eq. (26), we have

$$b - b_1 = \frac{4\alpha(x_s - L)}{(1 - \rho_1/\rho_0)}\left\{1 - \left[\frac{(\rho_0/\rho_1 - 1)}{(x_s - 1)}\,(x - L) + 1\right]^{-1/2}\right\}. \tag{33}$$

Applying the condition $u(x_s) = u_s$ to Eq. (32), we have

$$x_s - L = \frac{b_1}{4\alpha}\,\frac{(1 - \rho_1/\rho_0)}{(\rho_0^{1/2}/\rho_1^{1/2} - 1)}\left(\frac{u_1}{u_s} - 1\right). \tag{34}$$

However,

$$\frac{u_1}{u_s} = \frac{(u_s)_1}{u_s}\,\frac{u_1}{(u_s)_1} = \frac{(u_s)_1}{u_s}\,M_e\,,$$

and for an ideal gas at constant pressure

$$\frac{(u_s)_1}{u_s} = \left(\frac{T_1}{T_s}\right)^{1/2} \approx \left(\frac{\rho_0}{\rho_1}\right)^{1/2}.$$

Thus Eq. (34) can be written as

$$x_s - L = \frac{b_1}{4\alpha}\,\frac{(1 - \rho_1/\rho_0)}{(\rho_0^{1/2}/\rho_1^{1/2} - 1)}\left[\left(\frac{\rho_0}{\rho_1}\right)^{1/2} M_e - 1\right]. \tag{35}$$

The quantity b_1 can be computed from Eq. (20) by putting $x = L$:

$$b_1 \equiv b(L) \approx \frac{2\sqrt{2}}{\sqrt{\pi}}\,R + \frac{\sqrt{2}}{\sqrt{\pi}}\left(1 - \frac{4}{\pi}\right)R = 1.377\,R\,. \tag{36}$$

For cases where $\rho_0 >> \rho_1$ (which is always true for rocket exhaust plumes), using the value of b_1 from Eq. (36), Eqs. (35), (32) and (33) can be simplified to

$$\frac{x_s - L}{R} = 4.3\, M_e\,, \tag{37}$$

$$\frac{u_1}{u} = 1 + M_e \left\{ \left[\frac{0.232\ (\rho_0/\rho_1)}{M_e} \left(\frac{x - L}{R}\right) + 1 \right]^{1/2} - 1 \right\}, \tag{38}$$

and

$$\frac{b}{R} = 1.377 \left\{ 1 + M_e \left[1 - \left\{ \frac{0.232}{M_e} \left(\frac{\rho_0}{\rho_1}\right)\left(\frac{x - L}{R}\right) + 1 \right\}^{-1/2} \right] \right\}. \tag{39}$$

The decay of axial velocity and the spreading of the plume region are plotted in nondimensional form as shown in Fig. 11.

Anderson and Johns (10) measured the location x_s along the axis where the axial velocity becomes sonic downstream of five supersonic heated-air-jets and three types of solid-propellant rockets. Using their data on x_s for a different exit Mach number M_e, we can compute from Eq. (37) the length L of the undisturbed supersonic core. The results are given in Table 1 and Fig. 10.

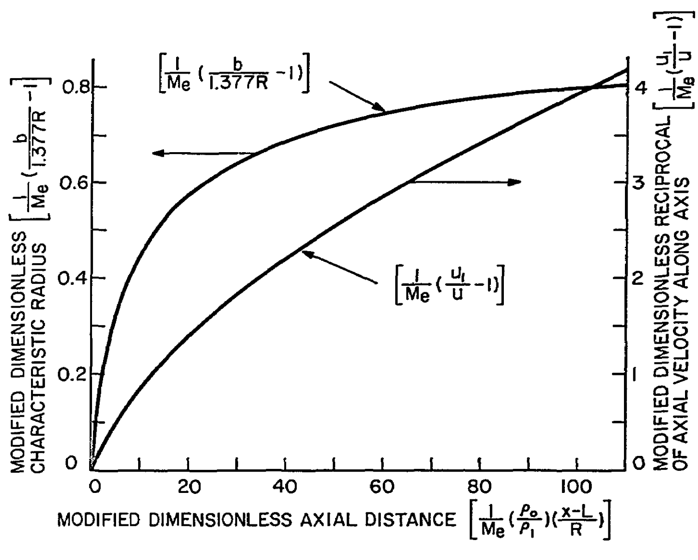

Fig. 11 - Theoretical predictions of the axial growth of the characteristic radius and the axial decay of the axial velocity for the region of a rocket exhaust plume between the end of an undisturbed supersonic core and the beginning of axial subsonic flow ($L \le x \le x_s$)

Table 1
Computed Length of the Supersonic Core from Data on the Axial Distance at Which the Axial Velocity Becomes Sonic, by Anderson and Johns (10)

Exhaust Source	Exit Mach Number M_e	Dimensionless Axial Distance where the Axial Velocity Becomes Sonic, $x_s/2R$	Computed Dimensionless Length of the Undisturbed Supersonic Core, L/R
Rocket 1	3.53	42.5	69.8
Rocket 2	2.82	29.0	45.9
Rocket 3	3.42	35.5	56.3
Air Jets			
740°F	1.40	9.5	13.0
1000°F	1.40	7.5	8.9
1500°F	1.40	7.0	8.0
1500°F	1.84	15.0	22.1
2200°F	1.40	8.5	11.0

Frauenberger and Forbister (9) also measured in their experiments the location along the axis of the exhaust plume of a solid-propellant rocket issuing into still air at which the axial velocity becomes sonic. From their reported data, the following information can be determined for their exhaust plume: $M_e = 3.22$; $R = 2.55$ in.; $(P^*)_s = 27.5$ psia = 12.8 psig [total pressure at $(x = x_s,\ r = 0)$]; $u_s = 2250$ ft/sec; and $x_s = 17.0$ ft. Using this information, we can compute the length L of the undisturbed supersonic core from Eq. (37): $L/R = 66.1$. This result is also plotted against M_e, as shown in Fig. 10.

From Fig. 10, we may conclude that from all available data, exceptionally good correlation on L/R against M_e has been established. Within the range of Mach numbers covered (up to $M_e = 3.5$), the following expression seems to hold:

$$\frac{L}{R} = \frac{80}{3}(M_e - 1). \tag{40}$$

Region of Fully Turbulent Subsonic Flow $(x \geq x_s)$

As shown in Fig. 3, the aerodynamic properties for flow in the region of fully turbulent subsonic flow are: axial velocity $u(x, r) = u(x)\exp(-r^2/b^2)$, where $u(x_s) = u_s$ and $b(x_s) = b_s$; density ρ_0; and pressure P_0. The total momentum flux is then

$$\begin{aligned} \overline{M} &= \int_0^\infty \rho_0 u^2(x)\exp(-2r^2/b^2)\, 2\pi r\, dr \\ &= \frac{\pi}{2}\rho_0 u^2 b^2 . \end{aligned} \tag{41}$$

The momentum flux equation then requires that

$$ub = u_s b_s . \tag{42}$$

The total mass flux is then

$$\overline{Q} = \int_0^\infty \rho_0\, u(x) \exp(-r^2/b^2)\, 2\pi r\, dr$$

$$= \pi\rho_0\, u(x)\, b^2(x) = \pi\rho_0\, u b^2 . \quad (43)$$

The mass flux equation can be approximated by

$$\frac{d\overline{Q}}{dx} = -2\pi\rho_0\, b v_b . \quad (44)$$

Again the entrainment assumption of Eq. (29) together with the expression for the total mass flux is introduced in Eq. (44), and we have

$$\frac{d}{dx}(ub^2) = 2\alpha b u . \quad (45)$$

Introducing Eq. (42) in Eq. (45), we have $db/dx = 2\alpha$; therefore

$$b(x) = 2\alpha(x - x_s) + b_s , \quad (46)$$

where the condition $b(x_s) = b_s$ has been used. Then from Eqs. (42) and (46), we have

$$u(x) = \frac{u_s b_s}{2\alpha(x - x_s) + b_s} . \quad (47)$$

Results for $b(x)$ and $u(x)$ are plotted against the axial distance from the point x_s where the axial velocity becomes sonic in a nondimensional fashion in Figs. 12 and 13, using the established value $\alpha = 0.08$.

Anderson and Johns (10) measured velocities at various axial stations downstream from the sonic velocity point of five supersonic heated-air jets and three types of solid-propellant rockets. Their experimental results are plotted against the theoretical result in Fig. 12.

CONCLUSIONS

The conclusions of this study can be summarized as follows:

1. The proposed aerodynamic model explains fairly satisfactorily the experimental measurements on various seemingly unrelated aerodynamic properties of some types of rocket exhaust plumes and the related supersonic air jet issuing into still ambient air.

2. The theoretical analysis based on the proposed aerodynamic model is developed from a rigorous set of governing aerodynamic equations and appropriate boundary conditions and therefore is valid throughout the total flow field of this type of rocket exhaust plume. Emphasis of the solutions is on the gross aerodynamic behavior rather than the minute details of the flow, and the solutions describe quite accurately the general structure of the plume as backed by various experimental evidence.

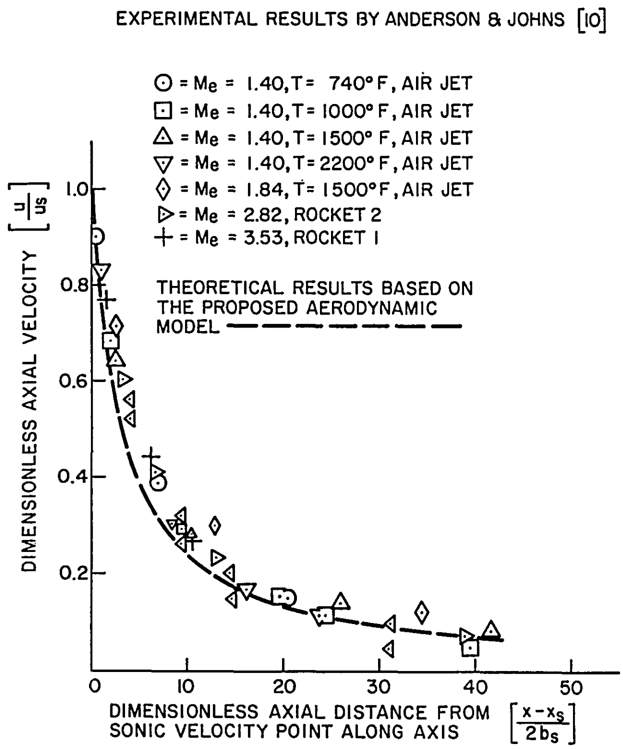

Fig. 12 - Decay of the axial velocity of the subsonic portion of the exhaust plume

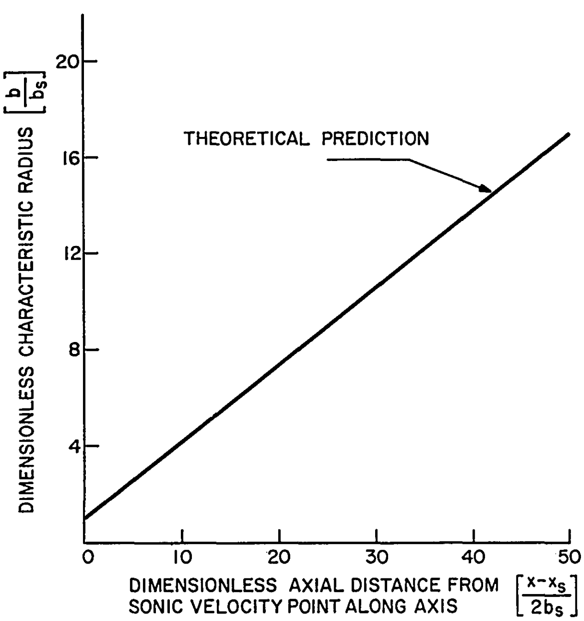

Fig. 13 - Spreading of the subsonic portion of the plume

3. The seemingly oversimplified geometry, especially in regions close to the nozzle exit, of the plume assumed in the proposed model approximates very closely the situation of a slightly overexpanded or underexpanded plume. For a plume from a rocket nozzle at an altitude slightly higher than the design altitude, the proposed aerodynamic model should still approximate the situation fairly closely for most of the flow field except for the region in the immediate neighborhood of the rocket nozzle exit, where the sudden expansion of the plume makes the appearance of the plume boundary apparently different from the one assumed in the proposed model in this region. However, detailed studies of this very region of a highly underexpanded plume have been reported at various places in the literature, such as the work by Smoot, Underwood, and Schroeder (2). The proposed aerodynamic model will then serve to analyze the flow field downstream from this region near the rocket nozzle.

4. This model should serve adequately for those relatively large number of cases where chemical reaction and two-phase flow do not exert a controlling influence. Improved models required for the high-energy propellant systems of today in which such parameters predominate are being developed.

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13. ABSTRACT
An aerodynamic model is proposed for the exhaust plume issuing from a rocket nozzle exit into quiescent ambient air in which chemical reaction or two-phase flow does not predominate. The model is based on the presently available knowledge and established aerodynamic principles for the various flow regimes involved. The major assumptions made are individually substantiated and justified by experimental evidence in existing literature.

A theoretical analysis is carried over the whole flow region based on the Kármán's integral approach in the hope of finding solutions for the gross aerodynamic behavior of the plume, especially for ranges up to the full region of the plume. Solutions are matched at the boundary of the inviscid core and turbulent mixing regions. Theoretical results agree closely with related experimental data.

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